

Geometry of silent chain - involute sprocket

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Abstract. Silent chains tend to have more and more usage in automotive industry. The study of these types of chains has a lot to consider due to the fact that silent chains are not standardized yet. The aim of this paper is to study and analyse the theoretical contact point between a silent chain's toothed plate and an involute sprocket. Varying different parameters of the chain and sprocket, there will be studied its effects on the theoretical contact point and the sprocket's profile geometry.

1 Introduction

Improving tribological performances of automotive power units can have significant benefits related to power increase, friction reduction, oil and fuel consumption reduction and not in the last place could increase also the reliability of engine components [1].

There are different approaches trying to reduce friction between automotive engine components, one of which is presented by [2], within there are considered different materials, surface treatments and surface coatings of engine timing chain transmissions. Chain transmissions due to their properties of high power and torque transmission, are used more often as component of internal combustion engine's timing system [3].

Within the group of chain transmissions used as timing chains, the silent chains besides the usual advantages of chain drives, has a reduced functioning noise, which tends to have a place between the necessary requirements of chain drives; a study that includes a few types of chain transmissions is presented by [4].

Improvements were done for the clearances compensation system of a timing chain with modelling, testing and geometric contact point optimization between the chain and the chain tensioning system for a bush chain in [5]. Geometric modelling of the contact point between the elements of a bush chain was established in [6].

A spring model for sprocket-chain load distribution was studied and then compared with traditional force-balance analysis in the work of [7]. Important facts were discussed about the pitch difference and its effect on the contact point that appears between the bushes of a bush chain with the chain sprocket [8]. These works contributed to a better understanding of the forces distribution and contact point position variation in case of bush and roller chains. Equations that describe the relation between the sprocket-silent chain link contacts were presented depending on the profile angle of the link's outer flange [9]. Influence of the centrifugal force and the number of teeth on the forces distribution that can be

found in a silent chain transmission was analysed in [10] with the conclusion that the load is distributed not just on the first links but on several links. Another view on the force distribution in the case of a silent chain transmission was presented with the consideration of the friction force that appears between the silent chain's elements and the sprocket in [11]. The sprocket constructions with straight tooth profile are replaced with curved flanks constructions. In this paper, the contact point position will be analysed for a silent chain transmission with involute sprocket.

2 Theoretical approach

The dynamic behaviour of the silent chain transmission is seemingly complex, regarding the corresponding contacts between the silent chain elements with the sprocket's teeth. Importance was given in this paper to the study of the contact point between the silent chain's toothed plate that transmits power to the corresponding involute surface of a teeth.

Figure 1 presents the theoretical contact point geometry for the silent chain- involute profile sprocket.

The aim of this geometric modelling is to determine the contact point between the silent chain's toothed plate and the sprocket's teeth of the chain transmission, and to define some of the basic parameters of a sprocket, using the known dimensions of the silent chain. For this there are taken into account the following input geometric parameters of the chain transmission: the pitch of the chain (p), the number of teeth of the sprocket (z), the pitch pressure angle (α_0), the angle of the toothed plate's outer flange line to the center line (α_x), the distance between the pin hole centre and the plate's flank (d_x) and the distance between the theoretical contact point and a line traced perpendicular to the toothed plate's outer flange through the centre of the pin hole (d_z). The output should be the module of the sprocket m and its profile displacement x .

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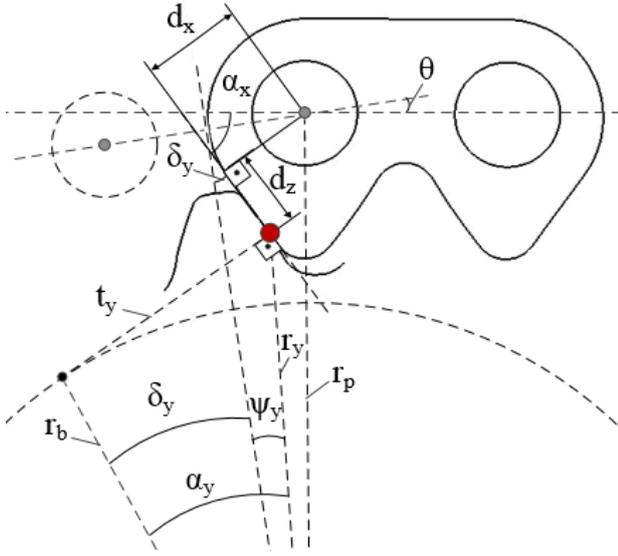


Fig. 1. Geometry of silent chain-sprocket contact.

The theoretical approach considers the definition of some basic parameters of the chain transmission:

- the angular pitch:

$$\Theta = \frac{360}{z}; \quad (1)$$

- the primitive module of the sprocket:

$$m_p = \frac{r_p}{z} = \frac{p}{z \cdot \sin\left(\frac{\Theta}{2}\right)}. \quad (2)$$

The standardized module [12] of the sprocket m should be smaller than the primitive module, calculated with (2).

Pitch radius of the sprocket is calculated with:

$$r = \frac{m \cdot z}{2}, \quad (3)$$

the base circle radius is calculated with:

$$r_b = r \cdot \cos \alpha_0, \quad (4)$$

the pitch radius of the silent chain transmission can be described as:

$$r_p = \frac{p}{\sin\left(\frac{\Theta}{2}\right)}. \quad (5)$$

The distance t_y results as

$$t_y = \sqrt{r_p^2 - (r_b + d_z)^2} - d_x. \quad (6)$$

The angle α_y can be calculated as

$$\alpha_y = \text{atan} \frac{t_y}{r_b}. \quad (7)$$

Using (5), (6) and (7) we obtain the radius of the circle where the contact point can be found:

$$r_y = \frac{r_b}{\cos \alpha_b} \quad (8)$$

The angle between the toothed plate outer flange and the line passing through the middle of the sprocket's tooth has the following formula based on Figure 1:

$$\delta_y = 90 - \alpha_x - \Theta. \quad (9)$$

The angle between the line linking the theoretical contact point with the centre of the base circle and the line passing through the middle of the sprocket teeth:

$$\psi_y = \alpha_y - \delta_y \quad (10)$$

According to [12] we can write:

$$\psi_y = \frac{S}{d} + \text{inv} \alpha_0 + \text{inv} \alpha_y, \quad (11)$$

$$S = m \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_0 \right), \quad (12)$$

where: S is the sprocket tooth thickness at the pitch circle, and x is the tooth's profile displacement.

Using (11) and (12) we obtain the formula for the profile displacement:

$$x = \frac{z(\psi_y - \text{inv} \alpha_0 + \text{inv} \alpha_y) - \frac{\pi}{2}}{2 \cdot \tan \alpha_0}. \quad (13)$$

The analysis should be also developed with the goal of finding the position of the contact point on the flank of the chain plate, described by dimension d_z . In this case, the profile displacement x is also considered an input. By using (10), (11) and (12), the angle α_y is determined.

With t_y determined from (7), the position of the contact point defined by dimension d_z can be described by the generalized Pythagorean Theorem and has the following form:

$$d_z = \sqrt{r_p^2 - (t_y + d_x)^2} - r_b. \quad (14)$$

3 Numerical analysis – Results

The geometric model for the definition of the silent chain – involute sprocket contact has been applied for different values of the known input parameters. These parameters are presented in Table 1.

The values of the calculated parameters using the input parameters and the equations presented in the first subsection of this paper are presented in Table 2. The theoretical profile displacement x for given values of d_z and for different values of the sprocket's teeth z is show in Figure 2, while depending of the plate flank angle α_x is shown on Figure 3.

Table 1. Input parameters.

Parameters	Unit	Value
p	mm	8
d_x	mm	3.7
d_z	mm	2
α_x	deg	55
α_0	deg	20
z	-	38

The following conclusion can be drawn from the results presented in the diagrams from Figures 2 and 3:

- The needed profile displacement x is increasing with the number of teeth and with decrease of the distance d_x on the chain plate; There could be issues of undercutting for small number of teeth and larger dimensions of the chain plate, d_x .
- The needed profile displacement x is increasing with the plate flank angle α_x ; As seen in Figure 3, the values of a valid plate flank angle are in a relatively narrow range, this is why this angle is very important to be set properly; If this angle is not in the range, involute sprockets may not be possible to use.

Figure 4 represents the evolution of the ratio between the contact point displacement d_z and the chain pitch depending on the number of teeth, for profile displacement $x = 0$.

Figure 5 presents the ratio between the contact point displacement d_z and the chain pitch depending on the profile displacement $x = 0$.

The distance d_z is increasing with the number of teeth but it decreases with the distance d_x and profile displacement x . It can prove to be very important if we impose a perfect laying of the chain plate on the teeth of the sprocket.

Table 2. Results.

Parameters	Unit	Value
Θ	deg	9.47368
m_p	mm	2.54938
m	mm	2.25
r	mm	42.75
r_b	mm	40.17185
t_y	mm	20.12854
α_y	deg	26.61365
r_y	mm	44.93257
δ_y	deg	25.52631
ψ_y	deg	1.08733
S_y	mm	1.70542
x	-	-0.03652

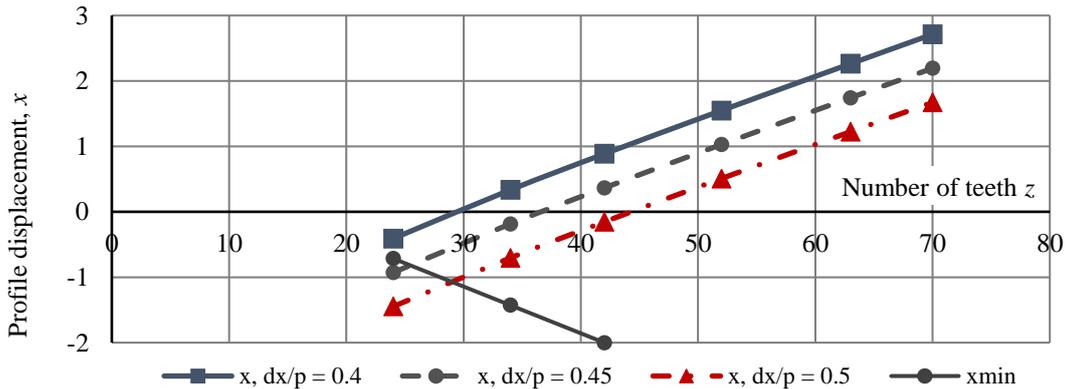


Fig. 2. Profile displacement depending on the number of teeth of sprocket.

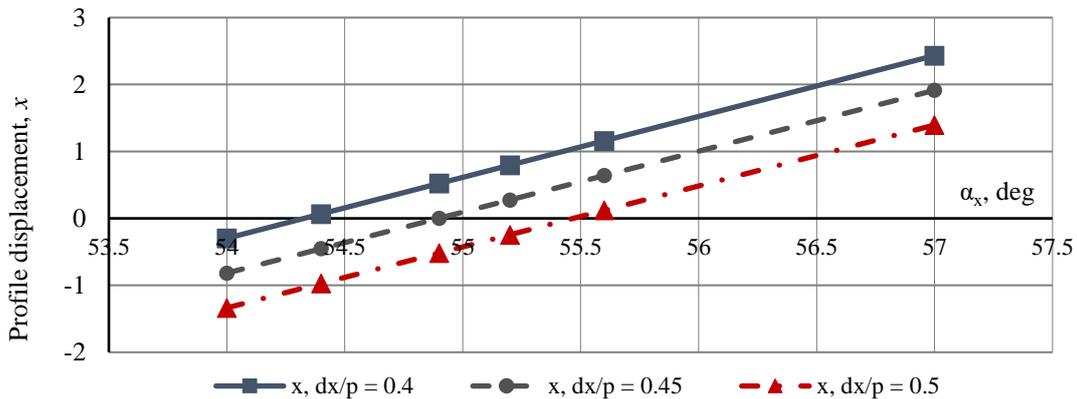


Fig. 3. Profile displacement depending on the angle of the flank of chain plate.

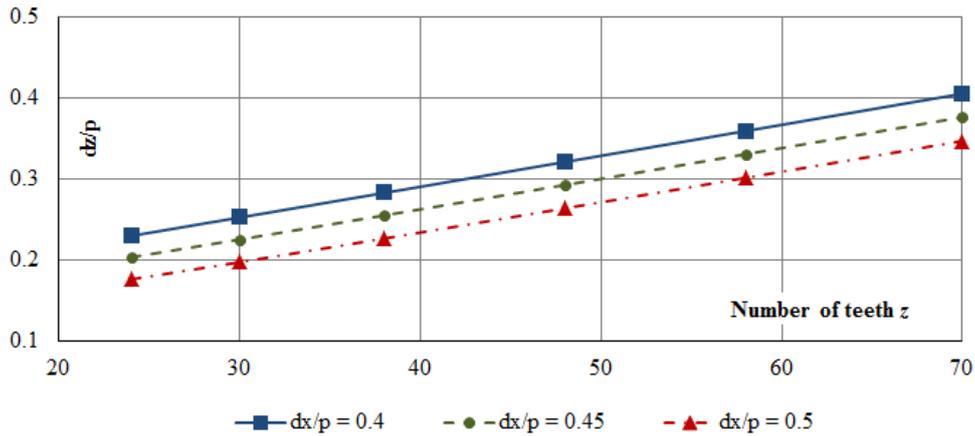


Fig. 4. Contact point displacement depending on number of teeth of the sprocket for $x = 0$.

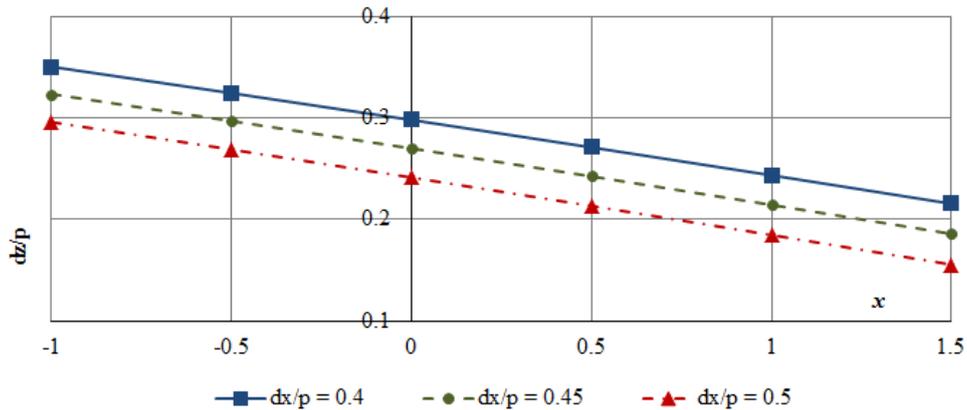


Fig. 5. Contact point displacement evolution over a given interval of the profile displacement of the sprocket.

3 Conclusion

The theoretical analysis starts with the presumption that during the functioning of the silent chain transmission the chain plate lays on the tooth without any sliding. This is not possible in the case of sprockets with straight tooth flank. This paper considers an involute sprocket. For this case, the perfect match between a chain and a sprockets leads to imposing a precise profile displacement. Even so, any lack of precision will create sliding between the tip of the plate and the tooth of the sprocket. Further proof of this can be found after a visual checking of the toothed links of the chain. They show on their outer flange marks of wear after use. Parameters like tooth flank angle, profile displacement of the sprocket have direct influence on the contact point's position and it's sliding over the involute flank of the sprocket's teeth.

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